AP Calculus BC Summer Assignment

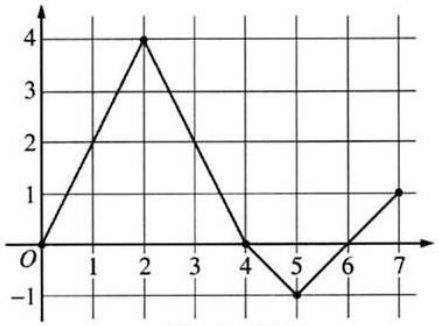
Name_____

I Abide by the Honor Code.

Due: Canvas submission by Friday, August 13, 2019 at 7:30 am.

ALL WORK MUST BE SHOWN NEATLY ON A SEPARATE DOCUMENT. CIRCLE ANSWERS. LATE PAPERS WILL BE PENALIZED.

No Calculator



Graph of f

1. Let f be the function defined on the closed interval [0, 7]. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

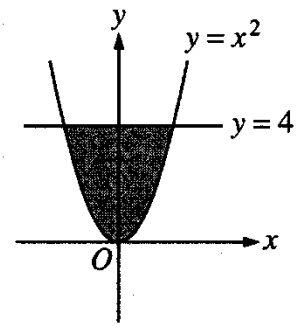
   (a) Find $g(3)$, $g'(3)$, and $g''(3)$.

   (b) Find the average rate of change of g on the interval $0 \le x \le 3$.

   (c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

   (d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

No Calculator



2. The shaded region *R* is bounded by the graph of $y = x^2$, and the line $y = 4$, as shown in the figure above.

   (a) Find the area of *R*.

   (b) Find the volume of the solid generated by revolving *R* about the *x*-axis.

   (c) There exists a number *k*, $k > 4$, such that when *R* is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of *k*.

Calculator Allowed

3. A particle moves along the *x*-axis so that its velocity at time *t*, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

   (a) Find the minimum acceleration of the particle.

   (b) Find the total distance traveled by the particle.

   (c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

No Calculator

4. A function f is continuous on the closed interval [-3, 3] such that $f(-3) = 4$ and $f(3) = 1$. The functions f' and f'' have the properties given in the table below.

x	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

(a) What are the x-coordinates of all the absolute maximum and absolute minimum points of f on the interval [-3, 3]? Justify your answer.

(b) What are the x-coordinates of all points of inflection of f on the interval [-3, 3]? Justify your answer.

(c) Sketch a graph that satisfies the given properties of f.

Calculator Allowed

5. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

(a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Justify your answer.

(b) Find the value of p such that the average value of f over the closed interval [-1, 2] is 1.

No Calculator

6. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\frac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

Calculator allowed

7. The volume V of a cone ($V = \frac{1}{3}\pi r^2 h$) is increasing at the rate of 28π cubic cm per second. At the instant when the radius r of the cone is 3 cm, its volume is 12π cubic cm and the radius is increasing at $\frac{1}{2}$ cm per second.

(a) At the instant when the radius of the cone is 3 cm, what is the rate of change of the area of its base?

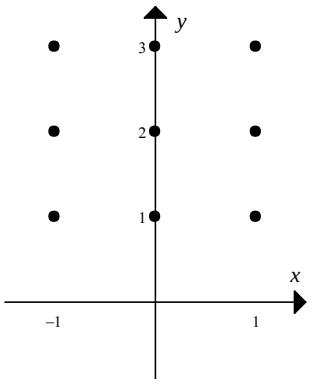
(b) At the instant when the radius of the cone is 3 cm, what is rate of change of its height h?

(c) At the instant when the radius of the cone is 3 cm, what is the instantaneous rate of change of the area of its base with respect to its height h?

No Calculator

8. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



(b) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.